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RESONANCE OSCILLATIONS OF A GAS IN A SHUT-END TUBE IN THE REGION OF TRANSITION TO SHOCK WAVES

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The authors describe theoretical and experimental studies of resonance oscillations of a gas in a tube, with one end shut and a periodically vibrating piston mounted on the other. Analytical expressions to calculate the amplitude of pressure fluctuations in a real gas and experimental results at the frequency of linear resonance are obtained. Theory shows good agreement with experiment. An experimental study is made of nonlinear resonances of the second and third orders and the transition from almost harmonic oscillations of the gas to highly nonlinear ones.

Resonance oscillations excited by harmonic motions of a piston in a tube with one end shut have been the subject of a number of investigations [1-10]. The most complete survey of the works is provided in [1]. As is known, in the vicinity of resonances a region of frequencies exists where periodic shock waves are formed. Resonance freqiencies are determined by the expression

$$\omega_{nm} = n\Omega/m \quad (n = 1\ ,\ m = 1, 2, 3)\ , \tag{1}$$

where $\omega_{11} = \Omega = \pi c_0/L$. At $m = 1$ we have natural frequencies of a gas column that determine the linear resonance, $m = 2$ corresponds to the second subharmonic resonance, and $m = 3$ to the third-order resonance (the third subharmonic resonance), respectively.

Resonance near the fundamental frequency is studied in most detail. Thus, in [2] a theory was suggested for the first time that describes well the form of a shock wave within the framework of the ideal-liquid model; a solution is obtained in the form of a function with zero mean value and a discontinuity characterizing the front of the shock wave. However, considerable discrepancy with experiment calls for taking account of losses. An analysis of the influence of losses on oscillations carried out in [3-5] is incomplete.

The second nonlinear resonance was investigated in [6-8], where oscillations were generated by a crank mechanism, and therefore the frequency spectrum of piston oscillations contained oscillations of doubled frequency. A theory of the second subharmonic resonance is suggested in [9].

The existence of the third nonlinear resonance was predicted in [10]. The third-order resonance has not been found experimentally.

All the mentioned works pertain to the case where the amplitude of the piston vibrations was sufficient for observation of shock waves. In the region of transition such studies were not conducted.

We seek to study the linear, second, and third nonlinear resonances, to obtain an analytical expression for calculating the amplitude of pressure fluctuations in a real liquid without employing experimental data in the region of linear resonance, to compare the obtained theoretical relations with experiment, and to carry out an experimental study of linear and nonlinear resonances in the region of transition to shock waves.

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Consider the Chester equation [2]

$$C-\frac{\varepsilon}{2}\sin\omega t=\left\{f(t)-\frac{2r}{\pi}\varepsilon^{1/2}\right\}^2-\frac{2\beta}{\kappa+1}\int_0^\infty f(t-\xi)\,\xi^{-1/2}\,d\xi\,, \tag{2}$$

where

$$\varepsilon=-\frac{4l}{(\kappa+1)\,L\cos\omega L/c_0}\,;\quad r=\frac{\pi c_0\,\mathrm{tg}\,(\omega L/c_0)}{(\kappa+1)\,\omega L\varepsilon^{1/2}}\,;$$
$$\beta=\frac{2}{R}\left\{\left(\frac{\nu}{\pi}\right)^{1/2}\left(1+\frac{\kappa-1}{\sqrt{\mathrm{Pr}}}\right)\right\}. \tag{3}$$

Neglecting the influence of viscosity and heat conduction ($\beta = 0$) and transforming the left-hand side of (2) with the substitution by $2\tau = \omega t + \pi/2$, we arrive at

$$\left(f(t)-\frac{2r}{\pi}\varepsilon^{1/2}\right)^2=\varepsilon\,(b^2+\cos^2\tau)\,, \tag{4}$$

where b is a new constant to be determined. According to [2], b is determined from the obvious condition of equality to zero of the average of the function $f(t)$ over a period. As a result, we have

$$f=\varepsilon^{1/2}\left\{\frac{2r}{\pi}\pm\cos\tau\right\}. \tag{5}$$

The sign in front of $\cos\tau$ changes each time that $\sin\tau = r$ [11]. We designate $h(t) = f(t - L/c_0)$ and differentiate (2) with respect to t:

$$l\omega\cos\omega t=(\kappa+1)\,Lh'\left\{h-(2r/\pi)\,\varepsilon^{1/2}\right\}-\beta L\int_0^\infty h'(t-\xi)\,\xi^{-1/2}d\xi\,. \tag{6}$$

The pressure on the piston is

$$p(t)=2c_0^2\,\rho_0 h(t)\,. \tag{7}$$

We multiply (6) by $(-\rho\omega/2\pi)$ and integrate with respect to t over an oscillation period:

$$-\frac{\omega}{\pi}\rho_0c_0^2\omega l\int_{t_{\mathrm{sh}}}^{t_{\mathrm{sh}}+2\pi/\omega}h(t)\cos\omega t\,dt=-\frac{\omega}{\pi}\rho_0c_0^2(\kappa+1)\,L\int_{t_{\mathrm{sh}}}^{t_{\mathrm{sh}}+2\pi/\omega}hh'\left(h-\frac{2r}{\pi}\varepsilon^{1/2}\right)dt+$$
$$+\frac{\omega}{\pi}\rho_0c_0^2\,\beta L\int_{t_{\mathrm{sh}}}^{t_{\mathrm{sh}}+2\pi/\omega}h(t)\int_0^\infty h'(t-\xi)\,\xi^{-1/2}\,d\xi\,, \tag{8}$$

where t_{sh} is the moment of the shock. It is easy to see that the term on the left is the work of the piston over the vibration period $\dot{E}_{\mathrm{p}}$, the first term on the right indicates the nonlinear losses $\dot{E}_{\mathrm{sh}}$ and the second term is the wall losses $\dot{E}_{\mathrm{w}}$ over the vibration period [11].

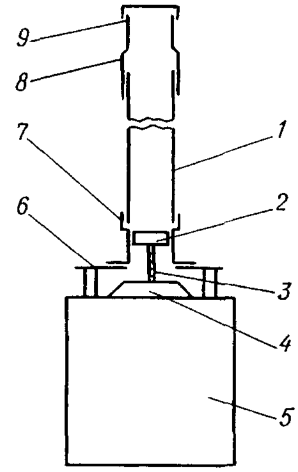

Fig. 1. Schematic of the experimental setup.

We consider how the viscosity influences the wave form, i.e., we will simultaneously consider the influence on the amplitude and the phase. Allowance for the integral in the right-hand side of (2) entails a change in (5). We can write [11]

$$f_0(\tau) = \varepsilon^{1/2}\left\{\frac{2r}{\pi} \pm \cos(\tau - \tau_0)\right\}. \tag{9}$$

Thus, the phase of occurrence of shocks will be displaced by τ_0 ($\Delta = \Delta + \tau_0$). If we introduce the so-called friction parameter [11]

$$s = \frac{2\beta}{\kappa + 1}\left(\frac{\pi}{\varepsilon\omega}\right)^{1/2}, \tag{10}$$

which is the ratio of the boundary-layer thickness to the tube radius divided by the square root of the Mach number of the piston, then for $s \ll 1$ and $\Delta_0 = 0$ we can obtain

$$\tau_0 = \frac{8s}{3\pi}\sin\frac{\pi}{4}. \tag{11}$$

We introduce $\tau^* = \tau - \tau_0$, and for exact resonance ($\Delta_0 = 0$) we have

$$h = h_0 \cos\tau^*, \quad \cos\omega t = -\sin(2\tau + 2\tau_0).$$

Then the equations for the work of the piston $\dot{E}_\text{p}$ and the wall losses $\dot{E}_\text{w}$ acquire the form

$$\dot{E}_\text{p} = \rho_0 c_0^3 h_0 a^* \varepsilon, \quad a^* = \frac{\kappa + 1}{2}\left(\cos 3\tau_0 + \frac{\cos 5\tau_0}{3}\right), \tag{12}$$

$$\dot{E}_\text{w} = \rho_0 c_0^3 h_0^2 b. \tag{13}$$

The nonlinear losses can be written as

$$\dot{E}_\text{sh} = \frac{2}{3}\rho_0 c_0^3 (\kappa + 1) h_1^3, \tag{14}$$

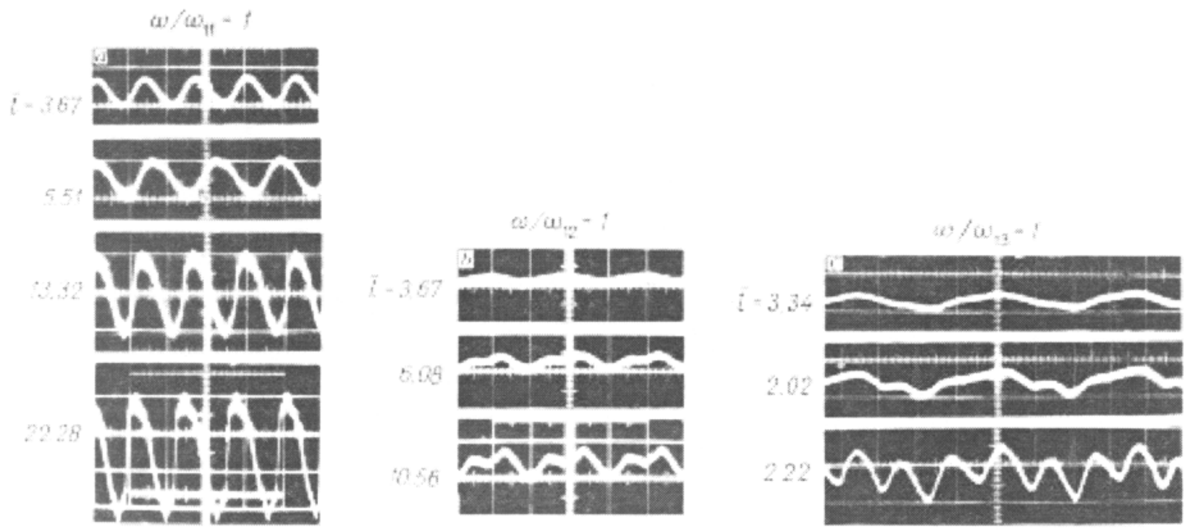

Fig. 2. Time variation of oscillograms of resonance oscillations with increase in the piston displacement amplitude: a) in the linear resonance ω_{11}; b) near the nonlinear resonance ω_{12}; c) near the nonlinear resonance ω_{13}.

where $h_1 = kh_0$, k is an as yet unknown constant that will be sought by an iteration method. For this, $h_0 = \varepsilon^{1/2}$ should be substituted into (12) and (13), $h_1 = k_1\varepsilon^{1/2}$ into (14). As a result, we have

$$k_1^3 = \frac{3}{4}\left(\cos 3\tau_0 + \frac{1}{3}\cos 5\tau_0\right) - \frac{3\pi}{2}\left(1 + \frac{\kappa - 1}{\sqrt{\Pr}}\right)\frac{1}{H(\kappa + 1)\,\varepsilon^{1/2}}. \tag{15}$$

In the next step, we assume that $h_0 = k_1\varepsilon^{1/2}$, $h_1 = k_2\varepsilon^{1/2}$ and obtain the following expression for k_2:

$$k_2^3 = \frac{3}{4}\left(\cos 3\tau_0 + \frac{\cos 5\tau_0}{3}\right)k_1 - \frac{3\pi}{2}\left(1 + \frac{\kappa - 1}{\sqrt{\Pr}}\right)\frac{1}{H(\kappa + 1)\,\varepsilon^{1/2}}k_1^2. \tag{16}$$

The process converges well for $s \leq 0.4$.

The experiment was conducted on the setup shown schematically in Fig. 1. Longitudinal oscillations of the gas column in closed tube 1 were generated by plane piston 2 connected by means of rod 3 to working table 4 of vibrator 5 of an electrodynamic bench, model VEDS-400. The use of the bench allowed a smooth variation in the frequency and amplitude of gas excitation. Glass tube 1 was hermetically connected to metallic cylinder 7, in which piston 2 vibrated. The cylinder was rigidly fastened via slab 7 to the body of bench 5. The other end of the glass tube was connected to metallic cylindrical head 8 for mounting piezoelectric pressure gauge 9. The entire length of the closed tube consisting of three parts was L = 870.5 mm. Its inside diameter was 39.3 mm. The pressure was measured by piezogauge 9, whose readings were recorded by an S1-54 oscillograph. The gas pressure measurement system was used previously in experiments [8, 12, 13]. The frequency and amplitude of the piston vibrations were regulated and measured by the control system of the bench. The excitation frequency $\omega/2\pi$ was varied from 60 to 210 Hz and monitored additionally by a Ch3-24 frequency meter.

Figure 2 shows oscillograms of resonance oscillations of the gas in relation to time with increase in the amplitude of piston displacement $\bar{l} = 10^4 l/L$ in the linear resonance ω_{11} (Fig. 2a) and near the nonlinear resonances ω_{12} (Fig. 2b) and ω_{13} (Fig. 2c).

We consider the results pertaining to the linear resonance near the first natural frequency $\omega_{11}/2\pi$ = 195.9 Hz. At a small excitation amplitude ($\bar{l}$ = 3.67) the gas oscillates almost by a harmonic law and the pres-

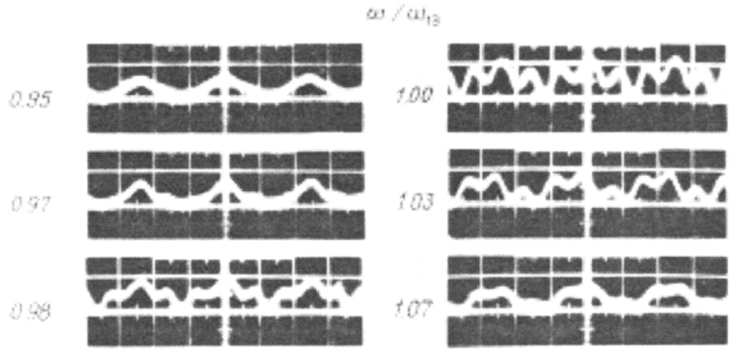

Fig. 3. Changes in oscillograms of gas pressure fluctuations in passing, with respect to the frequency, through the nonlinear resonance $\omega \sim \omega_{13}$ for $\bar{l} = 21.6$.

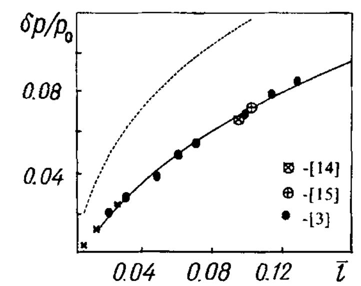

Fig. 4. Dimensionless pressure amplitude $\delta p/p_0$ versus relative amplitude of piston displacement $\bar{l}$.

sure wave has a symmetric and continuous form. A further increase in the excitation amplitude leads to deformation of the wave form; bends in the rarefaction and compression zones appear ($\bar{l} = 5.51$). The leading front of the wave between these zones becomes steep ($\bar{l} = 13.32$). At $\bar{l} = 22.28$ a strongly nonlinear wave close to a discontinuous one develops. As has been noted earlier [3, 4], at a small excitation amplitude, wall and volumetric losses dominate. With increase in amplitude, nonlinear effects become more pronounced, and the contribution of nonlinear losses increases. At large excitation amplitudes losses due to gas compression in nonlinear waves are the main reason for distortion of the amplitude and phase of the oscillations.

We consider the oscillograms of pressure oscillations in the region of nonlinear resonances $\omega \sim \omega_{12}$ and $\omega \sim \omega_{13}$ (Fig. 2b, c). It is seen that at a small excitation amplitude the gas oscillates almost by a harmonic law. With increase in excitation amplitude, the nonlinear behavior of the gas column is enhanced. Over a vibration period of the piston at $\omega = \omega_{12}$ two nonlinear waves appear, while at $\omega = \omega_{13}$ three waves are observed. In the first case, the wave is reflected twice from the shut end of the tube and the fundamental wave is followed by an intermediate wave (reflected from the piston) with a small pressure drop. In the second case, reflection occurs three times and we have the fundamental and two intermediate waves. Therefore for nonlinear resonances wave formation is less pronounced as compared to the linear resonance. In the latter case ($\omega = \omega_{11}$), the wave is reflected once from the shut end of the tube and therefore only the fundamental wave is observed (Fig. 2a). The maximum values of $\Delta\bar{p}$ at the resonance frequencies $\omega_{12}/\pi = 97.9$ Hz and $\omega_{13}/\pi = 65.3$ Hz were equal, respectively, to 0.52 and 0.299 ($\Delta\bar{p} = (p_2 - p_1)\cdot 10^2/p_0$, p_0 is the atmospheric pressure, p_2 and p_1 are the maximum and minimum pressures in a piston stroke).

Figure 3 provides oscillograms of the gas pressure fluctuations in passing, with respect to the frequency, through the nonlinear resonance $\omega \sim \omega_{13}$ at $\bar{l} = 21.6$. In the preresonance mode (at $\omega/\omega_{13} = 0.95$), the gas oscillations have a form close to harmonic. When the resonance is approached (at $\omega/\omega_{13} = 0.97$), we can observe sharp bends and one intermediate wave. Next (at $\omega/\omega_{13} = 0.98$), two intermediate waves are observed. In resonance (at $\omega/\omega_{13} = 1.00$), the amplitudes of the fundamental and intermediate waves increase to their maxima. After resonance, the wave amplitudes decrease (ω/ω_{13} = 1.03) and the gas oscillations acquire a form close to harmonic (ω/ω_{13} = 1.07).

In Fig. 4, the solid line shows the dimensionless pressure amplitude $\delta p/p_0$ as a function of the relative amplitude of piston displacement $\bar{l}$ calculated by the suggested theory for the parameters of the setup [3]. Here, the dashed line corresponds to calculation by the Chester theory (a nonviscous liquid) $\delta p/p_0 = 2\kappa\varepsilon^{1/2}$. The points depict results of [3, 14, 15], the crosses indicate results of the present experiment.

A comparison of the experiment with the theory of the second subharmonic resonance [9] when a perturbation initiated by the influence of the crank mechanism is disregarded has shown good agreement. A comparison of the experimentally obtained dimensionless amplitude of pressure fluctuations of the third-order resonance has not been carried out because of the absence of theory.

The experiments performed confirm that an increase in the excitation amplitude near linear and nonlinear resonances leads to the development of nonlinear effects and distortion of the form of the gas oscillations in a closed tube. Special features of formation of nonlinear pressure waves near a frequency threefold lower than the first natural frequency of the gas column have been revealed and investigated.

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NOTATION

ω_{11}, fundamental frequency of the gas column; c_0, velocity of sound in the undisturbed gas; L, tube length; ω, cyclic oscillation frequency; t, time; $\kappa = c_p/c_v$; l, amplitude of piston displacement; $\bar{l} = 10^4 l/L$, dimensionless amplitude of piston displacement; C, integration constant; R, tube radius; ν, kinematic-viscosity coefficient; $p(t)$, pressure on the piston; $t_{\rm sh}$, moment of shock in the shock wave; $\dot{E}_{\rm p}$, work of the piston over a period of the oscillations; $\dot{E}_{\rm sh}$, nonlinear losses; $\dot{E}_{\rm w}$, wall losses; s, friction parameter; Δ = arccos r; r, dimensionless frequency; $\delta p/p_0$, dimensionless pressure amplitude; $\Delta\bar{p}$, dimensionless pressure swing; Pr, Prandtl number; $H = R\sqrt{\omega/\nu}$, frequency parameter; ξ, independent integration constant. Subscripts: sh, shock; w, wall; p, piston; 0, average quantities over a period.

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